Spatially inhomogeneous modes of logistic equation with delay and small diffusion in a flat area

Sergey Glyzin[1,2], Vladimir Goryunov[1,2], Andrey Kolesov[1]

[1]P.G. Demidov Yaroslavl State University

[2]Scientific Center in Chernogolovka RAS

glyzin@uniyar.ac.ru, salkar@ya.ru, kolesov@uniyar.ac.ru

In the population biology, it is often used the logistic equation with delay and diffusion, which describes the change of the population density in some habitat with the simplest accounting of the age structure.

We considered the boundary value problem

$$\dot{N} = d\Delta N + r(1 - N_{t-1})N, \quad \left.\frac{\partial N}{\partial \nu}\right|_{\partial\Omega} = 0, \tag{1}$$

where $N = N(t,x) \in \mathbb{R}$ – population density; $N_{t-1} = N(t-1,x)$; $x \in \Omega \subset \mathbb{R}^2$; Δ – Laplace operator; d – diffusion coefficient; r – Malthusian coefficient of linear growth; ν – the direction of the outer normal to the border $\partial\Omega$ of bounded flat area Ω with measure equals to unity.

Provided that $r = \frac{\pi}{2} + \varepsilon$ and $d = \varepsilon\kappa$, where ε is a positive small parameter, we can apply to the boundary value problem (1) the asymptotic methods. It is possible to construct using these methods spatially inhomogeneous solutions of the problem (1) which are bifurcating at critical value of diffusion d_{crit} from spatially homogeneous stable cycle.

By increasing the parameter r we can obtain only numerical results. The main part of this work is devoted to their description. Spatially homogeneous cycle of (1) and spatially inhomogeneous cycles branching from it persist by all $r > \frac{\pi}{2}$, the amplitude of these cycles rise exponentially with growing of r, minimal average by x values of $N(t,x)$ tends to zero, and the average by t and x is equal to unity. At the same time, along with the available cycles, we showed numerically in a problem (1) the existence of spatially inhomogeneous modes with significantly different properties. In particular, fluctuations of the average by the area for these modes have minimal values separated from zero, while the average by area and time is greater than unity. These modes were called as modes with self-organization, since their qualitative characteristics in terms of population dynamics are significantly improved.

Numerical results, described by this work, carried out on the computing cluster of YSU (Laboratory of discrete and computational geometry) using parallel computing technologies.