Simulation of down-conversion processes in quantum circuit electrodynamics

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Over the last few years novel methods for the generation, manipulation and detecting of strongly correlated photons have been developed in area of quantum circuit electrodynamics [1]. As a main mechanism for generation of strong correlated photons – bi-photons or multi-photons – is usually used the down-conversion processes which have already been considered from phenomenological point of view in nonlinear optics [2]. Parametric down-conversion dynamics in quantum superconducting networks with incorporated Josephson junctions is considered as a promising phenomenon for generating entangled microwave photons. Such kind of photons are a key ingredient for quantum information processing and metrological applications.

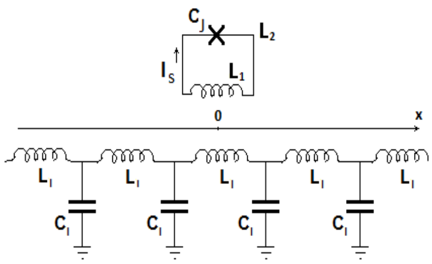

Fig.1. Discrete element model of transmission line coupled inductive with the loop containing a Josephson junction (JJ).

We investigate in the preset work the down-conversion processes in superconducting coplanar waveguide with an embedded superconducting loop, interrupted by a single Josephson junction (JJ). The model considered in this study is illustrated in figure 1. The quantized Hamiltonian of the system has a form

$$\hat{H} = \sum_k \hbar\omega_k \hat{b}_k^\dagger \hat{b}_k + \hbar\omega_J \hat{a}^\dagger \hat{a} - \frac{1}{3}\hbar\chi\left(\hat{a}+\hat{a}^\dagger\right)^3 + i\hbar g\left(\hat{a}-\hat{a}^\dagger\right)\hat{Q}_\mu + \frac{1}{2C_J}\hat{Q}_\mu^2 + \frac{c^2}{2L_\mu}\hat{\Phi}_\mu^2, \quad (1)$$

where the first term describes microwave photons in co-planar line, the second and the third terms are responsible for the Josephson junction, the fourth term is the interaction between them, and the last terms are some constrictions. Both of the constants g (coupling) and χ (nonlinearity) are defined by parameters of the system. The Heisenberg equations for photon field and Josephson operators are given by

$$\frac{d\hat{b}_k}{dt} = -i\omega_k \hat{b}_k + \tilde{c}_k\left(\frac{m\omega_k}{\hbar L_l}\hat{\Phi}_\mu + \frac{i}{2eJ}\left(\hat{p} - \frac{\hbar}{2e}\hat{Q}_\mu\right)\right),$$
$$\frac{d\hat{a}}{dt} = -i\omega_J \hat{a} + i\chi\left(\hat{a}+\hat{a}^\dagger\right)^2 - g\hat{Q}_\mu, \quad (2)$$

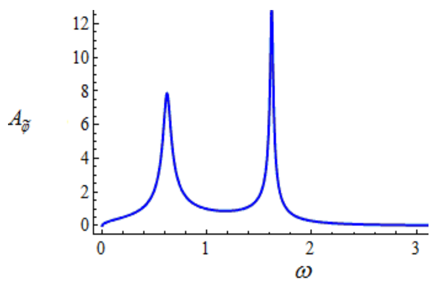

Fig. 2. The two-picks amplitude of Josephson oscillator as a function of frequency of external pulse (in AU).

where $\hat{p} = -i\sqrt{\hbar J\omega_J/2}\left(\hat{a}-\hat{a}^\dagger\right)$. The equations for bosonic field operators $\hat{b}_k$ are linear and may be solved exactly. Inserting this solution in second equation, we have got a nonlinear equation motion for the JJ operator $\hat{a}(t)$. To solve equation numerically we have used Fock basis $|n>$, which is defined by $\hat{a}^\dagger\hat{a}|n> = n|n>$. The infinite nonlinear system of equations for has been truncated and solved by using of the Runge-Kutta method. It is found that there is a region of system parameters when the down-conversion can be realized (see Fig. 2).

In summary, we have investigated analytically and numerically a realistic model of nonlinear element coupled with a microwave photons in co-planar superconducting waveguide. Our results demonstrate that in the system under the consideration takes place down-conversion processes for incoming photons.

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